Well-chosen method for an optimal design of doublet lens design

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Abstract: This paper presents a method for choosing a doublet design for the correction of longitudinal chromatic, spherical and coma aberrations. A secondary dispersion formula is utilized to sort out minimal longitudinal chromatic aberrations for the doublet. The program is developed with the Matlab software. An optimal doublet design to efficiently reduce both spherical aberration and coma will incorporate glass combination with a sufficiently large difference in the V-numbers and small powers. We succeed in obtaining an optimal doublet design with the proposed method.

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1. Introduction

Doublets are suitable for many applications where the field angle is limited to a few degrees. There have been a variety of methods for doublet design proposed by several researchers. Sharma has used the double-graph technique for the doublet designs [1]. Banerjee and Hazra studied the performance of genetic algorithms with respect to a relatively simple structural of lens design problem [2]. Robb has developed a method for the correction of axial color for at least three wavelengths using two different types of glass, and certain combinations may be found which are corrected at four and five wavelengths [3]. Sun has proposed an improved optimization method for doublet design using the shape factor of the third-order aberrations of a thick lens to compensate for high-order aberrations [4].

In this study, choosing the most appropriate glass for the doublet is the key. We use the equation for achromaticity based on a thin lens to determine what type of glass to use [5,6] after which we only need consider the spherical aberration and the coma of the Seidel aberration (for a thin lens). This is because the other aberrations are small enough that their effect on the field angle is only a few degrees. We need control only two aberrations to reduce the ray fan area, and thus obtain superior image quality.
2. Theory

In a lens design with a large field angle, seven different types of aberrations must be considered, including the longitudinal chromatic and lateral chromatic aberrations, spherical aberration, coma, astigmatism, field curvature and distortion. The doublet design must include two shape factors and two powers. Except for a fixed focal length lens, three different aberrations can be eliminated. To preserve image quality with a doublet design, it is only necessary to consider a small field angle. The longitudinal chromatic and spherical aberrations are independent of the field angle, but dependent on the pupil size. Thus we choose the longitudinal chromatic and spherical aberrations here. In addition, the coma aberration is also chosen because it is slightly been affected by the field angle.

The secondary spectrum equation can help to find an optimum value for the longitudinal chromatic aberration for all of the doublet glass combinations. In our approach it is suggested that the doublet that gives a large difference in the V-numbers and has a small power be chosen. This is the one that will lead to an efficient reduction of both spherical and coma aberrations. Finally, an optimal design of doublet can be obtained by the proposed method.

2.1 Achromatic glass combinations

Based on the human spectral response normalized to a maximum of one, as shown in Fig. 1, we choose the wavelengths of 460 nm, 555 nm and 647 nm, where 555 nm is the central wavelength. We define Abbe number as

\[ V_{555} = \frac{n_{555} - 1}{n_{460} - n_{647}}. \]  

(1)

We only consider SCHOTT glass data; the types of glasses are sorted according to the Abbe number [7].

![Human spectral response](image1.png)

Fig. 1. Human spectral response

The power \( K_{555} \) of a thin lens is the sum of the powers of its component surfaces. Hence for a single lens,

\[ K_{555} = (n_{555} - 1)(C_1 - C_2), \]  

(2)

where \( C_1 \) and \( C_2 \) represent the curvatures of the front and rear surfaces, respectively.

The primary color \( \delta \) is the difference in the power between the 460 nm light and 647 nm light [8,9].
\[ \delta = K_{460} - K_{647} = \frac{K_{555}}{V_{555}}. \]  

The partial dispersion coefficient \( P_{555,647} \) of a single lens is defined by

\[ P_{555,647} = \frac{n_{555} - n_{647}}{n_{460} - n_{647}}. \]

The secondary spectrum \( \varepsilon \) of a single lens is given by

\[ \varepsilon = K_{555} - K_{647} = \frac{P_{555,647}}{V_{555}} K_{555}. \]

Assume that the power \( K_{555} \) of the doublet can be calculated by

\[ K_{555} = (K_{555})_1 + (K_{555})_2, \]

where \((K_{555})_1\) and \((K_{555})_2\) are the powers of the first and the second lens, respectively.

The primary color \( \delta \) of the doublet is given by

\[ \delta = \delta_1 + \delta_2 = \frac{(K_{555})_1}{(V_{555})_1} + \frac{(K_{555})_2}{(V_{555})_2}, \]

where \( \delta_1 \) and \( \delta_2 \) are the primary color of the first and the second lens, respectively; \((V_{555})_1\) is the Abbe number of the first lens; \((V_{555})_2\) is the Abbe number of the second lens.

Under the condition of \( \delta = 0 \), we get

\[ (K_{555})_1 = \frac{(V_{555})_1}{(V_{555})_1 - (V_{555})_2} K_{555}, \]
\[ (K_{555})_2 = \frac{-(V_{555})_2}{(V_{555})_1 - (V_{555})_2} K_{555}. \]

The secondary spectrum \( \varepsilon \) is expressed as

\[ \varepsilon = \varepsilon_1 + \varepsilon_2 = \frac{(P_{555,647})_1 - (P_{555,647})_2}{(V_{555})_1 - (V_{555})_2} K_{555}, \]

where \( \varepsilon_1 \) is the secondary spectrum of the first lens; \( \varepsilon_2 \) is the secondary spectrum of the second lens; \((P_{555,647})_1\) is defined as the partial dispersion of the first lens; \((P_{555,647})_2\) is the partial dispersion of the second lens.

In this way, we can determine the glass data with the least amount of secondary spectrum, as shown in Table 1. Note that a smaller difference in partial dispersion and a larger difference in Abbe numbers indicate a good doublet glass combination with a good secondary spectrum.
### Table 1. SCHOTT glass data

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<th>Glass</th>
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<th>V&lt;sub&gt;555&lt;/sub&gt;</th>
<th>P&lt;sub&gt;555,647&lt;/sub&gt;</th>
<th>Glass</th>
<th>No.</th>
<th>V&lt;sub&gt;555&lt;/sub&gt;</th>
<th>P&lt;sub&gt;555,647&lt;/sub&gt;</th>
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### Glass Data Table

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### 2.2 Area of Longitudinal Chromatic Aberration

The relation between the secondary spectrum, partial dispersion and Abbe number is formulated as in Eq. (10). The relationship between partial dispersion and Abbe number is shown in Fig. 2. The set of glass combinations with the smallest difference in partial dispersions and the largest difference in Abbe numbers from the five sets is chosen to calculate the doublet glass combination. Figure 3 shows the glass combinations. Note that the focal length changes with the different wavelength. The vertical axis indicates the wavelength, the horizontal axis the focal point error, while point 0 indicates the focal point of the light source at the wavelength of 555 nm. Under the condition where the primary color is equal to zero, two focal points overlap (at the wavelengths of 460 nm and 647 nm). The area of longitudinal chromatic aberration is calculated from the area of the above curves, as shown in Table 2. We find that the area of the longitudinal chromatic aberration decreases as the secondary spectrum decreases. Thus the feature of longitudinal chromatic aberration can be judged by the value of the secondary spectrum.
Fig. 2. Partial dispersion vs. Abbe number

Fig. 3. Chromatic focal shift for the five glass combinations

Table 2. Data of the five glass combinations

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<th>Glass</th>
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<th>Axis chromatic aberration area ($\mu$m$^2$)</th>
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<td>0.0379</td>
<td></td>
</tr>
<tr>
<td>E 3: N-PK52A 34: N-KZFS2</td>
<td>0.2958</td>
<td>-0.1958</td>
<td>1.07</td>
<td>0.0314</td>
<td></td>
</tr>
</tbody>
</table>
2-3 Shape factor of the third-order spherical aberration and coma for a thin lens

The third-order spherical aberration and coma of a thin lens are defined below. The shape factor is expressed by \( \Lambda = h(C_1 + C_2) = \alpha_1 + \alpha_2 \), where \( h \) is the marginal ray height; \( \alpha \) is the curvature factor the defined by \( \alpha = hC \); \( U \) is defined as the conjugate factor \( U = u_1 + u_2 \); \( u_1 \) and \( u_2 \) represent the angles between the marginal ray and optical axis of a thin lens in object space and image space, respectively. The Lagrange invariance \( H = n_{555}(\vec{h}u - h\vec{u}) \), where \( \vec{h} \) and \( \vec{u} \) are the height and angle on the surface of the chief ray, respectively. For a simple expression of the third-order aberrations, the formula can be written as a function of shape factors, while \( S_I \) and \( S_{II} \) denote the values of the third-order spherical aberration and the coma. These terms are to be summed over all surfaces as usual. After some mathematical manipulation, we obtain the following aberrations [10]:

\[
S_I = \frac{1}{4} h^2 K \left[ \frac{hKn_{555}}{n_{555} - 1} + \frac{n_{555} + 2}{n_{555}} (\Lambda + U)^2 + 2U(\Lambda + U) \right], \tag{11}
\]

\[
S_{II} = -\frac{1}{2} hK_{555} H \left( \frac{2n_{555} + 1}{n_{555}} + \Lambda \left( \frac{n_{555} + 1}{n_{555}} \right) \right). \tag{12}
\]

The third-order spherical aberration and the coma for a doublet can be calculated by

\[
\sum S_I = S_{I1} + S_{I2}, \tag{13}
\]

\[
\sum S_{II} = S_{II1} + S_{II2}, \tag{14}
\]

where \( S_{I1} \) and \( S_{I2} \) are the third-order spherical aberrations of the first lens and the second lens, respectively; \( S_{II1} \) and \( S_{II2} \) indicate the coma aberrations for the first lens and the second lens, respectively.

If an object is at infinite distance, then the conjugate factor of both the first lens and the second lens are \( U_1 \) and \( U_2 \), respectively, which gives

\[
U_1 = -h(K_{555})_1, \tag{15}
\]

\[
U_2 = -h[(K_{555})_1 + K_{555}]. \tag{16}
\]

Since \( \Lambda_1 \) and \( \Lambda_2 \) are the shape factors for the first lens and the second lens, we substitute Eqs. (11), (12), (15) and (16) into Eqs. (13) and (14) to find the solution for the shape factor.

\[
\Lambda_1 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \tag{17}
\]

\[
\Lambda_2 = d\Lambda_1 + e, \tag{18}
\]

where the coefficients \( a \) to \( e \) are defined as

\[
a = \frac{h^2}{4} \left[ \left( \frac{n_1 + 2}{n_1} \right)(K_{555})_1 + \left( \frac{n_2 + 2}{n_2} \right) d^2 (K_{555})_2 \right].
\]
After obtaining the shape factor of the doublet lens, we determine the lens thickness. The following equations are used in the computer program:

\[
b = \frac{h^2}{4} \left[ \frac{4(n_1 + 1)}{n_1} U_1(K_{555})_1 + \frac{2(n_2 + 2)}{n_2} d(e(K_{555})_2) + \frac{4(n_2 + 1)}{n_2} d(K_{555})_2 U_2 \right]
\]

\[
c = \frac{h^2}{4} \left[ \frac{(3n_1 + 2)}{n_1} U_1^2(K_{555})_1 + \frac{(K_{555})_1^2 h^2 n_1^2}{(n_1 - 1)^2} + \frac{(n_2 + 2)}{n_2} e^2(K_{555})_2 U + \frac{4(n_2 + 1)}{n_2} U_2 e(K_{555})_2 + \frac{(3n_2 + 2)}{n_2} U_2^2(K_{555})_2 + \frac{(K_{555})_2^2 h^2 n_2^2}{(n_2 - 1)^2} \right] - \sum S_I
\]

\[
d = -\left( \frac{K_{555}}{n_1 + 1} \right) \left( n_2 + 1 \right) n_1
\]

\[
e = -\sum S_{II} - \frac{1}{2} h H \left( K_{555} \right) U_1 \left( \frac{2n_1 + 1}{n_1} \right) + \left( K_{555} \right) U_2 \left( \frac{2n_2 + 1}{n_2} \right)
\]

\[
e = \frac{1}{2} h H \left( K_{555} \right) \left( \frac{n_2 + 1}{n_2} \right)
\]

2.4 Thickening design of a doublet lens

After obtaining the shape factor of the doublet lens, we determine the lens thickness. The following equations are used in the computer program:

\[
h_1 K_{555} = h_1 (K_{555})_1 + h_2 (K_{555})_2 ,
\]

\[
h_1 (K_{555})_1 = [n_{555}] - 1 \left[ h_1 C_1 - h_2 C_2 \right] = [n_{555}] - 1 \left[ \alpha_1 - \alpha_2 \right].
\]

\[A_1 = h_1 C_1 + h_2 C_2 = \alpha_1 + \alpha_2 ,\]

\[h_3 (K_{555})_2 = [n_{555}] - 1 \left[ h_3 C_3 - h_4 C_4 \right] = [n_{555}] - 1 \left[ \alpha_3 - \alpha_4 \right].
\]

\[A_2 = h_3 C_3 + h_4 C_4 = \alpha_3 + \alpha_4 ,\]

where \( h_1, h_2, h_3 \) and \( h_4 \) are the marginal ray heights for each surface indicating the thicknesses of the doublet. Consider the stop arrangement at the first surface of the front lens. If the curvature factor is kept constant, then the powers and shape factor do not change during the lens thickening process. We use the paraxial ray formula to find the marginal ray height and the curvature for each surface. Finally, We obtain the finished optical doublet design.

3. Design procedure

Figure 4 shows a flow chart of the doublet design process. The design data for the initial stage are taken from \textit{Len Design Fundamentals} by Rudolf Kingslake [9]. The design parameters are effective focal length of 10mm; F-number of 3.333; and semi-field angle of 1°. The longitudinal chromatic aberration is first analyzed by looking at primary color and the secondary spectrum formulas. The glass combination needed to obtain smaller longitudinal chromatic aberrations and the power of each lens are found. Next, the shape factor of each surface can be calculated using the formula for third-order spherical aberration and coma for a thin lens. The curvature of each surface is found by the lens thickening process. Both the third-order spherical aberration and coma are controlled. The on-axis RMS spot size in the spot diagram should be less than the diffraction-limited value so that observed area of the
aberration curves in the 1.0 field meets the requirement of the spot diagram. If this condition is not satisfied, it is necessary to select an alternative glass combination. Finally, we obtain an optimal doublet design for which the on-axis RMS spot size is less than the value of the diffraction-limited, and ray aberration in the 1.0 field and longitudinal chromatic aberration are smaller. The sensitivity of the human eyes varies with the light wavelengths. The weighting factors for the three different wavelengths are defined in table 3. The area of the aberration curves from the ray-fan diagram is determined from the average area of the sum of the aberration curves at the above three wavelengths multiplied by the weighting factor.

Fig. 4. Flow chart for doublet design
Table 3. Weighting factors for three different wavelengths

<table>
<thead>
<tr>
<th>Wavelength</th>
<th>Weighting factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>460 nm</td>
<td>1</td>
</tr>
<tr>
<td>555 nm</td>
<td>2</td>
</tr>
<tr>
<td>647 nm</td>
<td>1</td>
</tr>
</tbody>
</table>

4. Design examples

Several design examples for the doublet are presented. A plot of the partial dispersion versus the Abbe number is shown in Fig. 5. The three sets of glass combinations are LITHOTEC-CAF 2 & P-SK57, LITHOTEC-CAF 2 & N-SF66 and N-PK51 & N-LASF31A. For comparison, K3 and F4 glass (chosen by Kingslake’s design) are used for the optimization design. To reduce longitudinal chromatic aberration, the chosen glass combinations must have smaller axis chromatic aberration area such as with LITHOTEC-CAF 2 & P-SK57. To reduce spherical aberration and coma, the glass selected should have a small lens power, that is to say, the difference between the relative Abbe numbers for the glass combination should be large, for example for the LITHOTEC-CAF2 & N-SF66 and N-PK51 & N-LASF31A doublet designs.

Table 4 shows a comparison of the glass combinations. The area of longitudinal chromatic aberration is the largest for the combination of K3 and F4. The area is the smallest for the combination of LITHOTEC-CAF2 & P-SK57. The largest lens power is obtained for the combination of LITHOTEC-CAF2 & P-SK57; the smallest power for LITHOTEC-CAF2 & N-SF66.

An optimization design data is taken from Kingslake’s lens data. The area of the longitudinal chromatic aberration and the on-axis RMS [4] in the spot diagram are shown in table 5. We calculate the radius of the Airy disk (according to the Airy Disk definition [11]) at a wavelength of 555 nm is 2.255x10⁻³ mm. The on-axis RMS in the spot diagram for the doublet design is 2.2481x10⁻³ mm, which is less than the radius of the Airy disk. However the area of the longitudinal chromatic aberration curve is the largest of all of the glass combinations. A look at table 6 shows that it has the largest difference in the area of the ray-fan curve for the above three wavelengths. The ray-fan diagram and on-axis spot diagram for the doublet are shown in Fig. 6(a). The modulation transfer function (MTF) is shown in Fig.
6(b). The MTF plot reveals the image quality. The on axis RMS of the 555 nm spot diagram at is less than the radius of the Airy disk for this glass combination. It is obvious that the poor on-axis aberration at other wavelengths, caused by the on-axis MTF of the white light cannot reach the diffraction-limited.

<table>
<thead>
<tr>
<th>Glass combination</th>
<th>((V_{555})_1)</th>
<th>((V_{555})_2)</th>
<th>((P_{555,647})_1)</th>
<th>((P_{555,647})_2)</th>
<th>((K_{555})_1)</th>
<th>((K_{555})_2)</th>
<th>(\epsilon(10^6 \text{ mm}^{-1}))</th>
<th>Axis chromatic aberration area ((\mu\text{m}^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>K3, F4</td>
<td>48.35</td>
<td>29.68</td>
<td>0.3685</td>
<td>0.3555</td>
<td>0.25901</td>
<td>-0.15901</td>
<td>70.00</td>
<td>0.4225</td>
</tr>
<tr>
<td>LITHOTEC-CAF2, P-SK57</td>
<td>78.11</td>
<td>48.92</td>
<td>0.3698</td>
<td>0.3699</td>
<td>0.267612</td>
<td>-0.16761</td>
<td>0.2217</td>
<td>0.0159</td>
</tr>
<tr>
<td>LITHOTEC-CAF2, N-SF66</td>
<td>78.11</td>
<td>16.68</td>
<td>0.3698</td>
<td>0.3394</td>
<td>0.127141</td>
<td>-0.02714</td>
<td>49.56</td>
<td>0.3029</td>
</tr>
<tr>
<td>N-PK51, N-LASF31A</td>
<td>63.15</td>
<td>33.19</td>
<td>0.3696</td>
<td>0.3602</td>
<td>0.21078</td>
<td>-0.11078</td>
<td>31.39</td>
<td>0.1899</td>
</tr>
</tbody>
</table>

Table 5. Doublet design for K3, F4

<table>
<thead>
<tr>
<th>Surf/Type</th>
<th>Radius (mm)</th>
<th>Thickness (mm)</th>
<th>Glass</th>
<th>Power ((\text{mm}^{-1}))</th>
<th>Axis chromatic aberration area ((\mu\text{m}^2))</th>
<th>On-axis RMS spot size ((555 \text{ nm})) ((\mu\text{m}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>OBJ</td>
<td>Inf</td>
<td>Inf</td>
<td></td>
<td>0.259</td>
<td>-0.169</td>
<td>0.4225</td>
</tr>
<tr>
<td>STOP 2</td>
<td>-3.5099</td>
<td>0.75</td>
<td>K3</td>
<td>0.259</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-3.5239</td>
<td>0.25</td>
<td>F4</td>
<td>0.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>-99.8583</td>
<td>9.2221</td>
<td></td>
<td>0.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IMA</td>
<td>Inf</td>
<td></td>
<td></td>
<td>0.25</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6. Ray fan area for doublet (K3, F4)

<table>
<thead>
<tr>
<th>semi angle</th>
<th>460 nm</th>
<th>555 nm</th>
<th>647 nm</th>
<th>460 nm - 647 nm</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Fan area(M+S)</td>
<td>Fan area(M+S)</td>
<td>Fan area(M+S)</td>
<td>Fan area(M+S)</td>
</tr>
<tr>
<td>0.7</td>
<td>4.608E-03</td>
<td>1.543E-03</td>
<td>3.015E-03</td>
<td>2.68E-03</td>
</tr>
<tr>
<td>0.7</td>
<td>6.188E-03</td>
<td>5.270E-03</td>
<td>5.727E-03</td>
<td>5.61E-03</td>
</tr>
<tr>
<td>0.7</td>
<td>8.135E-03</td>
<td>7.274E-03</td>
<td>7.480E-03</td>
<td>7.54E-03</td>
</tr>
</tbody>
</table>
Table 7 shows the data for the LITHOTEC-CAF2 and P-SK57 glass combinations. The chromatic aberration is excellent for all four glass combinations although it can be seen that the larger the lens power, the worst the aberration behavior; see table 8. It is obvious that the difference in area of ray-fan diagram for different wavelengths is small. However, it is the larger lens power with the worst aberration which leads to the worse MTF case, as shown in Fig. 7.

Table 7. Doublet design for LITHOTEC-CAF2, P-SK57

<table>
<thead>
<tr>
<th>Surf.:Type</th>
<th>Radius (mm)</th>
<th>Thickness (mm)</th>
<th>Glass</th>
<th>Power (mm(^{-1}))</th>
<th>Axis chromatic aberration area (µm(^2))</th>
<th>On-axis RMS spot size (555 nm) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OBJ</td>
<td>Inf</td>
<td>Inf</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>STOP</td>
<td>3.9546</td>
<td>0.75</td>
<td>1: LITHOTEC-CAF2</td>
<td></td>
<td>0.268</td>
<td>0.0159</td>
</tr>
<tr>
<td>2</td>
<td>-2.5983</td>
<td>0.0162</td>
<td></td>
<td></td>
<td>-0.177</td>
<td>0.1175</td>
</tr>
<tr>
<td>3</td>
<td>-2.6591</td>
<td>0.25</td>
<td>21: P-SK57</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>-13.6216</td>
<td>9.2879</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>IMA</td>
<td>Inf</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 8. Ray fan area for doublet (LITHOTEC-CAF2, P-SK57)

<table>
<thead>
<tr>
<th>Semi angle</th>
<th>460 nm</th>
<th>555 nm</th>
<th>647 nm</th>
<th>460 nm - 647 nm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fan area(M+S)</td>
<td>Fan area(M+S)</td>
<td>Fan area(M+S)</td>
<td>Fan area(M+S)</td>
</tr>
<tr>
<td>0</td>
<td>7.956E-02</td>
<td>7.453E-02</td>
<td>7.191E-02</td>
<td>7.52E-02</td>
</tr>
<tr>
<td>0.7</td>
<td>8.019E-02</td>
<td>7.511E-02</td>
<td>7.244E-02</td>
<td>7.58E-02</td>
</tr>
<tr>
<td>1</td>
<td>8.100E-02</td>
<td>7.572E-02</td>
<td>7.297E-02</td>
<td>7.64E-02</td>
</tr>
</tbody>
</table>
The LITHOTEC-CAF2 and N-SF66 glass combination is chosen for the doublet since this is the one with the largest difference in Abbe numbers and the small lens power of each lens. A look at table 9 reveals that the RMS of on-axis spot diagram is less than the value of the diffraction-limited. The area of the longitudinal chromatic aberration is 0.3029 µm². The glass combination which is ranked only lower than K3 and F4 is shown in table 4. Table 10 shows the areas of the ray-fan curves at three wavelengths. They have a large difference. The MTF plot is shown in Fig. 8. The on-axis MTF of a white light is less than the value of the diffraction-limited (for the large longitudinal chromatic aberration). The off-axis MTF is affected by the on-axis MTF and cannot reach the diffraction-limited.

### Table 9. Doublet design for LITHOTEC-CAF2, N-SF66

<table>
<thead>
<tr>
<th>Surf/Type</th>
<th>Radius (mm)</th>
<th>Thickness (mm)</th>
<th>Glass</th>
<th>Power (mm⁻¹)</th>
<th>Axis chromatic aberration area (µm²)</th>
<th>On-axis RMS spot size (555 nm) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OBJ</td>
<td>Inf</td>
<td>Inf</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>STOP</td>
<td>5.8178</td>
<td>0.75</td>
<td>LITHOTEC-CAF2</td>
<td>0.127</td>
<td>0.3029</td>
<td>1.9716E-03</td>
</tr>
<tr>
<td>2</td>
<td>-7.9738</td>
<td>0.0162</td>
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<td>-0.026</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-5.1206</td>
<td>0.25</td>
<td>N-SF66</td>
<td></td>
<td>0.3029</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>-6.1020</td>
<td>9.6499</td>
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<td></td>
</tr>
<tr>
<td>IMA</td>
<td>Inf</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 10. Ray fan area for doublet (LITHOTEC-CAF2, N-SF66)

<table>
<thead>
<tr>
<th>semi angle</th>
<th>460 nm</th>
<th>555 nm</th>
<th>647 nm</th>
<th>460 nm - 647 nm</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3.469E-03</td>
<td>2.858E-03</td>
<td>2.184E-03</td>
<td>2.84E-03</td>
</tr>
<tr>
<td>0.7</td>
<td>3.603E-03</td>
<td>3.189E-03</td>
<td>2.242E-03</td>
<td>3.06E-03</td>
</tr>
<tr>
<td>1</td>
<td>3.791E-03</td>
<td>3.571E-03</td>
<td>2.470E-03</td>
<td>3.35E-03</td>
</tr>
</tbody>
</table>
The RMS of the on-axis spot diagram for the last combination, N-PK51 and N-LASF31A, is much less than the value of the diffraction-limited. The area of the longitudinal chromatic aberration is less than that of the LITHOTEC-CAF2 and N-SF66 combination, as shown in Table 11. Table 12 shows the area of the ray-fan curve at three wavelengths. There is only a small difference. As seen in Fig. 9, both the on-axis MTF and off-axis MTF are very close to the value of the diffraction-limited.

Table 11. Lens Data for N-PK51, N-LASF31A

<table>
<thead>
<tr>
<th>Surf/Type</th>
<th>Radius (mm)</th>
<th>Thickness (mm)</th>
<th>Glass</th>
<th>Power (mm⁻¹)</th>
<th>Axis chromatic aberration area (µm²)</th>
<th>On-axis RMS spot size (555 nm) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OBJ</td>
<td>Inf</td>
<td>Inf</td>
<td></td>
<td></td>
<td></td>
<td>1.3768E-03</td>
</tr>
<tr>
<td>STOP</td>
<td>5.4628</td>
<td>0.75</td>
<td>4:N-N-PK51</td>
<td>0.211</td>
<td>0.1899</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-4.4337</td>
<td>0.0162</td>
<td></td>
<td>-0.115</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-4.6074</td>
<td>0.25</td>
<td>61:N-LASF31A</td>
<td>-0.115</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>-11.6623</td>
<td>9.4531</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IMA</td>
<td>Inf</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 12. Ray fan area for doublet (N-PK51, N-LASF31A)

<table>
<thead>
<tr>
<th>Semi angle</th>
<th>460 nm Fan area(M+S)</th>
<th>555 nm Fan area(M+S)</th>
<th>647 nm Fan area(M+S)</th>
<th>460 nm - 647 nm Fan area(M+S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.302E-03</td>
<td>1.317E-03</td>
<td>1.890E-03</td>
<td>1.46E-03</td>
</tr>
<tr>
<td>0.7</td>
<td>1.577E-03</td>
<td>2.254E-03</td>
<td>2.394E-03</td>
<td>2.12E-03</td>
</tr>
<tr>
<td>1</td>
<td>2.096E-03</td>
<td>3.028E-03</td>
<td>2.946E-03</td>
<td>2.80E-03</td>
</tr>
</tbody>
</table>
5. Conclusion

An optimal design for a doublet includes a combination of different types of glass chosen for both the small area of the longitudinal chromatic aberration curve and lower power of each lens. The area of the longitudinal chromatic aberration curve can be controlled by the partial dispersion of the lens. In this study, we find a well-chosen method for glass combination of a doublet design. A good design has a larger difference in the Abbe numbers and small lens powers of each lens.

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