Design and Correction of Optical Systems

Lecture 10: Correction principles I

2017-06-16

Herbert Gross

Summer term 2017
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<td>07.04.</td>
<td>Basics</td>
<td>Law of refraction, Fresnel formulas, optical system model, raytrace, calculation approaches</td>
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<tr>
<td>2</td>
<td>21.04.</td>
<td>Materials and Components</td>
<td>Dispersion, anormal dispersion, glass map, liquids and plastics, lenses, mirrors, aspheres, diffractive elements</td>
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<td>3</td>
<td>28.04.</td>
<td>Paraxial Optics</td>
<td>Paraxial approximation, basic notations, imaging equation, multi-component systems, matrix calculation, Lagrange invariant, phase space visualization</td>
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<td>05.05.</td>
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<td>Pupil, ray sets and sampling, aperture and vignetting, telecentricity, symmetry, photometry</td>
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<td>Longitudinal and transverse aberrations, spot diagram, polynomial expansion, primary aberrations, chromatical aberrations, Seidels surface contributions</td>
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<td>6</td>
<td>19.05.</td>
<td>Wave Aberrations</td>
<td>Fermat principle and Eikonal, wave aberrations, expansion and higher orders, Zernike polynomials, measurement of system quality</td>
</tr>
<tr>
<td>7</td>
<td>26.05.</td>
<td>PSF and Transfer function</td>
<td>Diffraction, point spread function, PSF with aberrations, optical transfer function, Fourier imaging model</td>
</tr>
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<td>8</td>
<td>02.06.</td>
<td>Further Performance Criteria</td>
<td>Rayleigh and Marechal criteria, Strehl definition, 2-point resolution, MTF-based criteria, further options</td>
</tr>
<tr>
<td>9</td>
<td>09.06.</td>
<td>Optimization and Correction</td>
<td>Principles of optimization, initial setups, constraints, sensitivity, optimization of optical systems, global approaches</td>
</tr>
<tr>
<td>10</td>
<td>16.06.</td>
<td>Correction Principles I</td>
<td>Symmetry, lens bending, lens splitting, special options for spherical aberration, astigmatism, coma and distortion, aspheres</td>
</tr>
<tr>
<td>11</td>
<td>23.06.</td>
<td>Correction Principles II</td>
<td>Field flattening and Petzval theorem, chromatical correction, achromate, apochromate, sensitivity analysis, diffractive elements</td>
</tr>
<tr>
<td>12</td>
<td>30.06.</td>
<td>Optical System Classification</td>
<td>Overview, photographic lenses, microscopic objectives, lithographic systems, eyepieces, scan systems, telescopes, endoscopes</td>
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<tr>
<td>13</td>
<td>07.07.</td>
<td>Special System Examples</td>
<td>Zoom systems, confocal systems</td>
</tr>
</tbody>
</table>
1. Introduction
2. Sensitivity and improvement process
3. Special options for spherical aberration
4. Astigmatism
5. Symmetry
6. Coma
7. Distortion
8. Aspheres and higher orders
System Design Phases

1. Paraxial layout:
   - specification data, magnification, aperture, pupil position, image location
   - distribution of refractive powers
   - locations of components
   - system size diameter / length
   - mechanical constraints
   - choice of materials for correcting color and field curvature

2. Correction/consideration of Seidel primary aberrations of 3rd order for ideal thin lenses, fixation of number of lenses

3. Insertion of finite thickness of components with remaining ray directions

4. Check of higher order aberrations

5. Final correction, fine tuning of compromise

6. Tolerancing, manufactability, cost, sensitivity, adjustment concepts
- Existing solution modified
- Literature and patent collections
- Principal layout with ideal lenses
  successive insertion of thin lenses and equivalent thick lenses with correction control

- Approach of Shafer
  AC-surfaces, monochromatic, buried surfaces, aspherics
- Expert system
- Experience and genius
Initial Conditions

Valid for object in infinity:

1. Total refractive power

$$s_1 = -\infty$$

$$F' = \sum_{m=1}^{M} \omega_m \sum_{n=1}^{N} F'_{nm}$$

2. Correction of Seidel aberrations

2.1 Dichromatic correction of marginal ray

axial achromatical

$$F' = \sum_{m=1}^{M} \omega_m^2 \sum_{n=1}^{N} \frac{F'_{nm}}{V_{nm}}$$

2.2 Dichromatic correction of chief ray

achromatical lateral magnification

$$\bar{V} = \sum_{m=1}^{M} \omega_m \omega_{pm} \sum_{n=1}^{N} \frac{F'_{nm}}{V_{nm}}$$

2.3 Field flattening

Petzval

$$\frac{F'}{n} = \sum_{m=1}^{M} \sum_{n=1}^{N} \frac{F'_{nm}}{n_{nm}}$$

2.4 Distortion correction according

to Berek

$$0 = \sum_{m=1}^{M} \omega_{pm} \sum_{n=1}^{N} F'_{nm}$$

3. Tri-chromatical correction

Secondary spectrum

$$\frac{F'P}{V} = \sum_{m=1}^{M} \omega_m^2 \sum_{n=1}^{N} \frac{P_{nm}F'_{nm}}{V_{nm}}$$
Number of Lenses

- Approximate number of spots over the field as a function of the number of lenses. Linear for small number of lenses. Depends on mono-/polychromatic design and aspherics.

- Diffraction limited systems with different field size and aperture.
## Correction Effectiveness

- Effectiveness of correction features on aberration types

<table>
<thead>
<tr>
<th>Aberration</th>
<th>Primary Aberration</th>
<th>5th Aberration</th>
<th>Chromatic Aberration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spherical Aberration</td>
<td>a</td>
<td>e</td>
<td>f</td>
</tr>
<tr>
<td>Coma</td>
<td>(c)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Astigmatism</td>
<td></td>
<td>(g)</td>
<td></td>
</tr>
<tr>
<td>Petzval Curvature</td>
<td></td>
<td></td>
<td>(i)</td>
</tr>
<tr>
<td>Distortion</td>
<td></td>
<td></td>
<td>(j)</td>
</tr>
<tr>
<td>5th Order Spherical</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Axial Color</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lateral Color</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Secondary Spectrum</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spherochromatism</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Lens Parameters

- Lens Bending
- Power Splitting
- Power Combination
- Distances
- Stop Position

### Action

- Refractive Index
- Dispersion
- Relative Partial Disp.
- GRIN
- Cemented Surface
- Aplanatic Surface
- Aspherical Surface
- Mirror
- Diffractive Surface
- Symmetry Principle
- Field Lens

### Impact Ratings

- Makes a good impact.
- Makes a smaller impact.
- Makes a negligible impact.
- Zero influence.

Ref: H. Zügge
Usefull options for accelerating a stagnated optimization:

- split a lens
- increase refractive index of positive lenses
- lower refractive index of negative lenses
- make surface with large spherical surface contribution aspherical
- break cemented components
- use glasses with anomalous partial dispersion
Operationen with zero changes in first approximation:

1. Bending a lens.
2. Flipping a lens into reverse orientation.
3. Flipping a lens group into reverse order.
4. Adding a field lens near the image plane.
5. Inserting a powerless thin or thick meniscus lens.
6. Introducing a thin aspheric plate.
7. Making a surface aspheric with negligible expansion constants.
8. Moving the stop position.
9. Inserting a buried surface for color correction, which does not affect the main wavelength.
10. Removing a lens without refractive power.
11. Splitting an element into two lenses which are very close together but with the same total refractive power.
12. Replacing a thick lens by two thin lenses, which have the same power as the two refracting surfaces.
13. Cementing two lenses a very small distance apart and with nearly equal radii.
Structural Changes for Correction

- **Lens bending**

- **Lens splitting**

- **Power combinations**

- **Distances**

Ref: H. Zügge
- Removal of a lens by vanishing of the optical effect
- For single lens and cemented component
- Problem of vanishing index: Generation of higher orders of aberrations

1) adapt second radius of curvature
2) shrink thickness to zero

a) Geometrical changes: radius and thickness

b) Physical changes: index
Sensitivity of a System

- Sensitivity/relaxation:
  Average of weighted surface contributions of all aberrations

- Correctability:
  Average of all total aberration values

- Total refractive power
  \[ F = F_1 + \sum_{j=2}^{k} \omega_j F_j \]

- Important weighting factor: ratio of marginal ray heights
  \[ \omega_j = \frac{h_j}{h_1} \]
Quantitative measure for relaxation

\[ A_j = \omega_j \cdot \frac{F_j}{F} = \frac{h_j \cdot F_j}{h_1 \cdot F} \]

with normalization

\[ \sum_{j=1}^{k} A_j = 1 \]

Non-relaxed surfaces:
1. Large incidence angles
2. Large ray bending
3. Large surface contributions of aberrations
4. Significant occurrence of higher aberration orders
5. Large sensitivity for centering

Internal relaxation can not be easily recognized in the total performance

Large sensitivities can be avoided by incorporating surface contribution of aberrations into merit function during optimization
Sensitivity of a System

Representation of wave
Seidel coefficients $[\lambda]$

Double Gauss 1.4/50

Ref: H.Zügge
Relaxed System

- Example: achromate with cemented/splitted setup
- Equivalent performance
- Inner surfaces of splitted version more sensitive

(a) Cemented achromate $f=100$ mm, NA = 0.1

(b) Splitted achromate $f=100$ mm, NA = 0.1

Ref: H. Zügge
Sensitivity and Relaxation

- Reality:
  - as-designed performance: not reached in reality
  - as-built-performance: more relevant

- Possible criteria:
  1. Incidence angles of refraction
  2. Squared incidence angles
  3. Surface powers
  4. Seidel surface contributions
  5. Permissible tolerances

- Special aspects:
  - relaxed systems does not contain higher order aberrations
  - special issue: thick meniscus lenses
Further Parameter of Sensitivity

- Possible further criteria for modified merit function to obtain relaxed systems
  1. cos-Γ-factor of ray bending

\[ \Gamma_j = n_{j'} \cdot s_j \cdot e_j - n_j \cdot \bar{s}_j \cdot \bar{e}_j \]

2. Squared sum of incidence angles

\[ \theta = \sqrt{\frac{1}{2N} \cdot \sum_{j=1}^{N} (i_{j'}^2 + i_j^2)} \]

Target: minimum value for \( i, i' \)

3. Optimization of performance and performance change simultaneously

\[ D = \sqrt{\sum_{m=1}^{M} \left( \frac{\partial M}{\partial p_m} \cdot \Delta p_m \right)^2} \]
As Built Performance

- Relaxed system: as built performance improved
- Typically:
  - no or weak correlation to designed performance
  - weak decrease in nominal performance possible

[Graph showing performance, nominal with tolerances, and best as-built performance with field size and average performance sensitivity axes.]
### Design Solutions and Sensitivity

- **Focussing 3 lens with NA = 0.335**
- **Spherical correction with/without compensation**
- **Red surface: main correcting surface**
- **Counterbeding every lens in one direction**

<table>
<thead>
<tr>
<th>counterbending</th>
<th>Dspot</th>
<th>SPH-min</th>
<th>SPH-max</th>
</tr>
</thead>
<tbody>
<tr>
<td>no</td>
<td>10.9</td>
<td>0.63</td>
<td>3.7</td>
</tr>
<tr>
<td>L1 +</td>
<td>0.38</td>
<td>4</td>
<td>151</td>
</tr>
<tr>
<td>L1 -</td>
<td>0.28</td>
<td>12</td>
<td>105</td>
</tr>
<tr>
<td>L2 -</td>
<td>0.19</td>
<td>14</td>
<td>95</td>
</tr>
<tr>
<td>L2 +</td>
<td>0.65</td>
<td>4</td>
<td>292</td>
</tr>
<tr>
<td>L3 -</td>
<td>0.18</td>
<td>5</td>
<td>151</td>
</tr>
<tr>
<td>L3 +</td>
<td>0.50</td>
<td>5</td>
<td>151</td>
</tr>
</tbody>
</table>
Microscopic Objective Lens

- Incidence angles for chief and marginal ray
- Aperture dominant system
- Primary problem is to correct spherical aberration
Photographic lens

- Incidence angles for chief and marginal ray
- Field dominant system
- Primary goal is to control and correct field related aberrations: coma, astigmatism, field curvature, lateral color
Symmetrical Dublet

- Variable focal length
  \( f = 15 \ldots 200 \text{ mm} \)

- Invariant:
  \( \text{object size } y = 10 \text{ mm} \)
  \( \text{numerical aperture } NA = 0.1 \)

- Type of system changes:
  - dominant spherical for large \( f \)
  - dominant field for small \( f \)

- Data:

<table>
<thead>
<tr>
<th>No</th>
<th>focal length [mm]</th>
<th>Length [mm]</th>
<th>spherical ( c_3 )</th>
<th>field curvature ( c_4 )</th>
<th>astigmatism ( c_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>200</td>
<td>808</td>
<td>3.37</td>
<td>-2.01</td>
<td>-2.27</td>
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<tr>
<td>2</td>
<td>100</td>
<td>408</td>
<td>1.65</td>
<td>1.19</td>
<td>-4.50</td>
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<tr>
<td>3</td>
<td>50</td>
<td>206</td>
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<td>3.45</td>
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<td>3.93</td>
<td>2.31</td>
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<tr>
<td>5</td>
<td>15</td>
<td>59</td>
<td>0.20</td>
<td>16.7</td>
<td>-5.33</td>
</tr>
</tbody>
</table>
- Effect of bending a lens on spherical aberration
- Optimal bending:
  - Minimize spherical aberration
- Dashed: thin lens theory
  - Solid: think real lenses
- Vanishing SPH for n=1.5
  - only for virtual imaging
- Correction of spherical aberration possible for:
  1. Larger values of the magnification parameter |M|
  2. Higher refractive indices

Ref: H. Zügge
Aplanatic Surfaces with Vanishing Spherical Aberration

- **Aplanatic surfaces**: zero spherical aberration:
  1. Ray through vertex
     \[ s' = s = 0 \]
  2. concentric
  3. Aplanatic

- **Condition for aplanatic surface**:
  \[ r = \frac{ns}{n + n'} = \frac{n' s'}{n + n'} = \frac{ss'}{s + s'} \]

- **Virtual image location**

- **Applications**:
  1. Microscopic objective lens
  2. Interferometer objective lens

- Linear growing aberrations with deviation of object location
- No shift invariance in z
Aplanatic lenses

- Combination of two spherical corrected surfaces: one concentric and one aplanatic surface: zero contribution of the whole lens to spherical aberration

- Not useful:
  1. aplanatic-aplanatic
  2. concentric-concentric

bended plane parallel plate, nearly vanishing effect on rays
Aplanatic Lenses

- Impact of aplanatic lenses in microscopy on magnification

\[ m = \frac{y'}{y} = \frac{ns'}{n's} \]

- Three cases of typical combinations

\[
\begin{align*}
A-C & : \quad m_{AC} = n \\
A-V & : \quad m_{AV} = n^2 \\
A-C & : \quad A-V \quad m = n^3
\end{align*}
\]
- Example lens
- Aperture NA = 0.5
- Spherical correction with one surface
Microscope Objective Lens

- Examples of large-working-distance objective lenses
- Aplanatic-concentric shell-lenses in the front group
- Large diameter of the lens
- Correction of spherical aberration: Splitting of lenses

- Distribution of ray bending on several surfaces:
  - smaller incidence angles reduces the effect of nonlinearity
  - decreasing of contributions at every surface, but same sign

- Last example (e): one surface with compensating effect

Ref: H. Zügge
Splitting of lenses and appropriate bending:
1. compensating surface contributions
2. Residual zone errors
3. More relaxed setups preferred, although the nominal error is larger

Ref: H. Zügge
- Better correction for higher index
- Shape of lens / best bending changes from
  1. nearly plane convex for n = 1.5
  2. meniscus shape for n > 2

Ref: H. Zügge
Correcting spherical aberration by cemented doublet:

- Strong bended inner surface compensates
- Solid state setups reduces problems of centering sensitivity
- In total 4 possible configurations:
  1. Flint in front / crown in front
  2. bi-convex outer surfaces / meniscus shape
- Residual zone error, spherical aberration corrected for outer marginal ray

<table>
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<tr>
<th>Configuration</th>
<th>Diagram</th>
<th>Aberration</th>
</tr>
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<tbody>
<tr>
<td>Crown in front</td>
<td><img src="a" alt="Diagram" /></td>
<td><img src="b" alt="Diagram" /></td>
</tr>
<tr>
<td>Flint in front</td>
<td><img src="c" alt="Diagram" /></td>
<td><img src="d" alt="Diagram" /></td>
</tr>
</tbody>
</table>

Ref : H. Zügge
Better correction for high index also for multiple lens systems

Example: 3-lens setup with one surface for compensation
Residual aberrations is quite better for higher index

Ref: H. Zügge
IR Objective Lens

- Aperture f/1.5
- Spectral 8 - 12 μm
- 2 lenses aspherical
Compound Systems

- System groups:
  1. Afocal zoom telescope
  2. Scanning group
  3. Reimager
- Suitable for cooled matrix detector
- Bending effects astigmatism
- For a single lens 2 bending with zero astigmatism, but remaining field curvature

![Diagram showing astigmatism and Seidel coefficients](image-url)

Ref: H. Zügge
**Principle of Symmetry**

- Perfect symmetrical system: magnification \( m = -1 \)
- Stop in centre of symmetry
- Symmetrical contributions of wave aberrations are doubled (spherical)
- Asymmetrical contributions of wave aberration vanishes \( W(-x) = -W(x) \)
- Easy correction of:
  - coma, distortion, chromatical change of magnification

![Diagram showing front and rear parts of a symmetrical optical system](image-url)
Symmetrical Systems

Ideal symmetrical systems:
- Vanishing coma, distortion, lateral color aberration
- Remaining residual aberrations:
  1. spherical aberration
  2. astigmatism
  3. field curvature
  4. axial chromatical aberration
  5. skew spherical aberration
Symmetry Principle

- Application of symmetry principle: photographic lenses
- Especially field dominant aberrations can be corrected
- Also approximate fulfillment of symmetry condition helps significantly: quasi symmetry
- Realization of quasi-symmetric setups in nearly all photographic systems

Ref: H. Zügge
Concentric system of Offner:

\[ d_1 = d_2 = \frac{r_1}{2} = r_2 \]

Due to symmetry:
Perfect correction of field aberrations in third order
- Catadioptric system with \( m = -1 \) according Dyson
  - Advantage: flat field
  - Application: lithography and projection
- Relation:
  \[
  r_L = \frac{n - 1}{n} \cdot r_M
  \]
- Residual aberration: astigmatism
Mono-Centric Systems

Offner

Offner-Wynne

Dyson

do-nothing lens
(only Petzval)

Sutton ball-lens

Retrofocus I

Retrofocus II
Mono-Centric Lenses

Schwarzschild

Newton

Stamenov

Schmidt

Stop

M1

focal plane (curved)

N-BK7

N-F2

Mono-Centric Lenses

Schwarzschild

Newton

Stamenov

Schmidt

Stop

M1

focal plane (curved)
Coma Correction: Achromat

- Bending of an achromate
  - optimal choice: small residual spherical aberration
  - remaining coma for finite field size
- Splitting achromate:
  - additional degree of freedom:
  - better total correction possible
  - high sensitivity of thin air space
- Aplanatic glass choice:
  vanishing coma

<table>
<thead>
<tr>
<th>Achromat bending</th>
<th>Image height:</th>
<th>y' = 0 mm</th>
<th>y' = 2 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pupil section:</td>
<td>meridional</td>
<td>meridional</td>
<td>sagittal</td>
</tr>
<tr>
<td>Transverse Aberration:</td>
<td>0.05 mm</td>
<td>0.05 mm</td>
<td>0.05 mm</td>
</tr>
</tbody>
</table>

Ref: H. Zügge
- Effect of lens bending on coma
- Sign of coma: inner/outer coma

From: H. Zügge
- Perfect coma correction in the case of symmetry
- But magnification \( m = -1 \) not useful in most practical cases

## Coma Correction: Symmetry Principle

<table>
<thead>
<tr>
<th>Symmetry principle</th>
<th>Image height: ( y' = 19 \text{ mm} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pupil section:</td>
<td>meridional</td>
</tr>
<tr>
<td>Transverse Aberration:</td>
<td>( \Delta y' )</td>
</tr>
<tr>
<td></td>
<td>0.5 mm</td>
</tr>
</tbody>
</table>

(a) 

(b) 

From: H. Zügge
### Coma Correction: Stop Position and Aspheres

- Combined effect, aspherical case prevent correction

<table>
<thead>
<tr>
<th>Plano-convex element exhibits spherical aberration</th>
<th>Sagittal coma $\Delta y'$ 0.5 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
</tr>
<tr>
<td><img src="image3.png" alt="Image" /></td>
<td><img src="image4.png" alt="Image" /></td>
</tr>
<tr>
<td><img src="image5.png" alt="Image" /></td>
<td><img src="image6.png" alt="Image" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Spherical aberration corrected with aspheric surface</th>
<th>Sagittal coma $\Delta y'$ 0.5 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image7.png" alt="Image" /></td>
<td><img src="image8.png" alt="Image" /></td>
</tr>
<tr>
<td><img src="image9.png" alt="Image" /></td>
<td><img src="image10.png" alt="Image" /></td>
</tr>
<tr>
<td><img src="image11.png" alt="Image" /></td>
<td><img src="image12.png" alt="Image" /></td>
</tr>
</tbody>
</table>

Ref: H. Zügge
Distortion and Stop Position

- Sign of distortion for single lens: depends on stop position and sign of focal power
- Ray bending of chief ray defines distortion
- Stop position changes chief ray height at the lens

<table>
<thead>
<tr>
<th>Lens</th>
<th>Stop location</th>
<th>Distortion</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>positive</td>
<td>rear</td>
<td>V &gt; 0</td>
<td>tele photo lens</td>
</tr>
<tr>
<td>negative</td>
<td>in front</td>
<td>V &gt; 0</td>
<td>loupe</td>
</tr>
<tr>
<td>positive</td>
<td>in front</td>
<td>V &lt; 0</td>
<td>retrofocus lens</td>
</tr>
<tr>
<td>negative</td>
<td>rear</td>
<td>V &lt; 0</td>
<td>reversed binocular</td>
</tr>
</tbody>
</table>

Ref: H. Zügge
Coexistence of Aberrations: Balance

- Example: Achromate
- Balance:
  1. Zonal spherical
  2. Spot
  3. Secondary spectrum

Ref: H. Zügge
Coexistence of Aberrations : Balance

- Example: Apochromate
- Balance:
  1. zonal spherical
  2. Spot
  3. Secondary spectrum