Design and Correction of Optical Systems

Lecture 4: Optical systems
2017-05-05
Herbert Gross
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<th>Date</th>
<th>Topic</th>
<th>Notes</th>
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<td>1</td>
<td>07.04.</td>
<td>Basics</td>
<td>Law of refraction, Fresnel formulas, optical system model, raytrace, calculation approaches</td>
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<tr>
<td>2</td>
<td>21.04.</td>
<td>Materials and Components</td>
<td>Dispersion, anomalous dispersion, glass map, liquids and plastics, lenses, mirrors, aspheres, diffractive elements</td>
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<td>3</td>
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<td>Paraxial Optics</td>
<td>Paraxial approximation, basic notations, imaging equation, multi-component systems, matrix calculation, Lagrange invariant, phase space visualization</td>
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<td>Pupil, ray sets and sampling, aperture and vignetting, telecentricity, symmetry, photometry</td>
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<td>Longitudinal and transverse aberrations, spot diagram, polynomial expansion, primary aberrations, chromatic aberrations, Seidel's surface contributions</td>
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<td>Wave Aberrations</td>
<td>Fermat principle and Eikonal, wave aberrations, expansion and higher orders, Zernike polynomials, measurement of system quality</td>
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<td>26.05.</td>
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<td>Diffraction, point spread function, PSF with aberrations, optical transfer function, Fourier imaging model</td>
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<td>8</td>
<td>02.06.</td>
<td>Further Performance Criteria</td>
<td>Rayleigh and Marechal criteria, Strehl definition, 2-point resolution, MTF-based criteria, further options</td>
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<td>9</td>
<td>09.06.</td>
<td>Optimization and Correction</td>
<td>Principles of optimization, initial setups, constraints, sensitivity, optimization of optical systems, global approaches</td>
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<td>10</td>
<td>16.06.</td>
<td>Correction Principles I</td>
<td>Symmetry, lens bending, lens splitting, special options for spherical aberration, astigmatism, coma and distortion, aspheres</td>
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<tr>
<td>11</td>
<td>23.06.</td>
<td>Correction Principles II</td>
<td>Field flattening and Petzval theorem, chromatic correction, achromate, apochromate, sensitivity analysis, diffractive elements</td>
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<tr>
<td>12</td>
<td>30.06.</td>
<td>Optical System Classification</td>
<td>Overview, photographic lenses, microscopic objectives, lithographic systems, eyepieces, scan systems, telescopes, endoscopes</td>
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<tr>
<td>13</td>
<td>07.07.</td>
<td>Special System Examples</td>
<td>Zoom systems, confocal systems</td>
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</table>
1. Basic definitions
2. Ray sampling
3. Pupil
4. Vignetting
5. Telecentricity
6. Miscellaneous imaging aspects
7. Photometry of optical systems
- Imaging on axis: circular / rotational symmetry
  only spherical aberration and chromatical aberrations

- Finite field size, object point off-axis:
  - chief ray as reference
  - skew ray bundles: coma and distortion
  - Vignetting, cone of ray bundle not circular symmetric
  - to distinguish: tangential and sagittal plane
Numerical Aperture and F-number

- Classical measure for the opening: numerical aperture
  \[ NA' = n \cdot \sin u' \]

- In particular for camera lenses with object at infinity: F-number
  \[ F\# = \frac{f}{D_{EnP}} \]

- Numerical aperture and F-number are to system properties, they are related to a conjugate object/image location

- Paraxial relation
  \[ F\# = \frac{1}{2n' \tan u'} \]

- Special case for small angles or sine-condition corrected systems
  \[ F\# = \frac{1}{2NA'} \]
Generalized F-Number

- More general definition of the F-number for systems with finite object location

- Effective or working F-number with

\[ s' = f'(1 - m) \]

and

\[ \sin u' = \frac{D_{Exp}}{2s'} = \frac{D_{Exp}}{2f'(1 - m)} = \frac{m_p D_{EnP}}{2n' f'(1 - m)} \]

we get as a relation with the object-in-infinity-case

\[ F_{#}^{\text{eff}} = \frac{1}{2n' \sin u'} = F_{#} \cdot \frac{1 - m}{m_p} \]

m is the system magnification, \( m_p \) is the pupil magnification
Special rays in 3D

- Meridional rays: in main cross section plane
- Sagittal rays: perpendicular to main cross section plane
- Coma rays: Going through field point and edge of pupil
- Oblique rays: without symmetry
Tangential- and Sagittal Plane

- Off-axis object point:
  1. Meridional plane / tangential plane / main cross section plane contains object point and optical axis
  2. Sagittal plane:
     perpendicular to meridional plane through object point
Ray Fan and Ray Cone

- Ray fan:
  2-dimensional plane set of rays

- Ray cone:
  3-dimensional filled ray cone
- Pupil sampling for calculation of tranverse aberrations:
  all rays from one object point to all pupil points on x- and y-axis

- Two planes with 1-dimensional ray fans

- No complete information: no skew rays
- Pupil sampling in 3D for spot diagram: all rays from one object point through all pupil points in 2D
- Light cone completely filled with rays
Pupil Sampling

- Criteria:
  1. iso energetic rays
  2. good boundary description
  3. good spatial resolution

- Options:
  - polar grid
  - cartesian
  - isoenergetic circular
  - statistical
  - hexagonal
  - pseudo-statistical (Halton)
  - Fibonacci spirals
- Artefacts due to regular gridding of the pupil of the spot in the image plane
- In reality a smooth density of the spot is true
- The line structures are discretization effects of the sampling

Pupil Sampling Spot Artefacts

cartesian  hexagonal  statistical
The physical stop defines the aperture cone angle $u$

The real system may be complex

The entrance pupil fixes the acceptance cone in the object space

The exit pupil fixes the acceptance cone in the image space

Ref: Julie Bentley
Entrance and Exit Pupil

- Entrance pupil
- Exit pupil
- Upper marginal ray
- Upper coma ray
- Lower coma ray
- Lower marginal ray
- Chief ray
- Stop
- Field point of image
- Outer field point of object
- On axis point of object
- Object point on axis
Relevance of the system pupil :

- Brightness of the image
  Transfer of energy

- Resolution of details
  Information transfer

- Image quality
  Aberrations due to aperture

- Image perspective
  Perception of depth

- Compound systems:
  matching of pupils is necessary, location and size
Optical Image formation:

- Sequence of pupil and image planes
- Matching of location and size of image planes necessary (trivial)
- Matching of location and size of pupils necessary for invariance of energy density
- In microscopy known as Köhler illumination
Pupil Mismatch

- Telescopic observation with different f-numbers
- Bad match of pupil location: key hole effect

\[ F\# = 2.8 \]
\[ F\# = 8 \]
\[ F\# = 22 \]

a) pupil adapted

b) pupil location mismatch

Ref: H. Schlemmer
Mismatch of Eyepieces

Pupil mismatch

Eye relief, spherical aberration, eye movement

Ref: Smith, Ceragioli, Berry, Telescopes, Eyepieces, Astrographs, Willman-Bell, 2012
Field Stop Images

- Iris blades

Ref.: V. Blahnik
Vignetting

- Artificial vignetting:
  Truncation of the free area of the aperture light cone

- Natural Vignetting:
  Decrease of brightness according to $\cos^4 w$ due to oblique projection of areas and changed photometric distances
- 3D-effects due to vignetting
- Truncation of the at different surfaces for the upper and the lower part of the cone
- Truncation of the light cone with asymmetric ray path for off-axis field points
- Intensity decrease towards the edge of the image
- Definition of the chief ray: ray through energetic centroid
- Vignetting can be used to avoid uncorrectable coma aberrations in the outer field
- Effective free area with extrem aspect ratio: anamorphic resolution
Vignetting

- Photographic lens 100mm f/2

1. with strong vignetting
rays aimed to boundaries

2. Without vignetting
no really transmitted rays shown

Ref: V. Paruchuru
Vignetting

- Illumination fall off in the image due to vignetting at the field boundary
Pupil Sphere

- Pupil sphere: equidistant sine-sampling

\[
\sin(U) \quad \sin(U')
\]

angle $U$ non-equidistant

\[
\text{equidistant } \sin(U)
\]

\[
y_o \quad y'
\]

\[
pupil sphere
\]

\[
\text{object entrance pupil}
\]

\[
\text{exit pupil}
\]

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Aplanatic system:
- Sine condition fulfilled
- Pupil has spherical shape
- Normalized canonical coordinates for pupil and field

\[ \bar{y}_p = \frac{y_p}{h_{EnP}} \]
\[ \bar{y}'_p = \frac{y'_p}{h'_{ExP}} \]
\[ \bar{y} = \frac{n \cdot \sin u}{\lambda} \cdot y \]
\[ \bar{y}' = \frac{n' \cdot \sin u'}{\lambda} \cdot y' \]
Telecentricity

- Special stop positions:
  1. stop in back focal plane: object sided telecentricity
  2. stop in front focal plane: image sided telecentricity
  3. stop in intermediate focal plane: both-sided telecentricity

- Telecentricity:
  1. pupil in infinity
  2. chief ray parallel to the optical axis
• Double telecentric system: stop in intermediate focus
• Realization in lithographic projection systems
### Infinity cases

- **Systematic of all infinity cases**
- **Physically impossible:**
  1. object and entrance pupil in infinity
  2. image and exit pupil in infinity

<table>
<thead>
<tr>
<th>case</th>
<th>object</th>
<th>image</th>
<th>entrance pupil</th>
<th>exit pupil</th>
<th>example</th>
<th>sample layout</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>finite</td>
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<td>finite</td>
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<td>infinity object telecentric</td>
<td>infinity image telecentric</td>
<td>lithographic projection lens 4f-system</td>
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<td>finite</td>
<td>afocal zoom telescopes beam expander</td>
<td><img src="image4.png" alt="afocal zoom telescopes" /></td>
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<td>infinity</td>
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<td>finite</td>
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<td>camera lens focusing lens</td>
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<td>infinity</td>
<td>finite</td>
<td>finite</td>
<td>eyepiece collimator</td>
<td><img src="image7.png" alt="eyepiece collimator" /></td>
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<tr>
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<td>infinity</td>
<td>infinity object telecentric</td>
<td>finite</td>
<td>microscopic lens</td>
<td><img src="image8.png" alt="microscopic lens" /></td>
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<td>finite</td>
<td>infinity image telecentric</td>
<td>infinity metrology lens</td>
<td><img src="image9.png" alt="infinity metrology lens" /></td>
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<td>impossible</td>
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<td>infinity</td>
<td>impossible</td>
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</tr>
</tbody>
</table>
Anamorphotic Imaging Setup

- Anamorphic imaging:
  different magnifications in x- and y-cross section, tangential and sagittal magnification

- Identical image location in both sections

- Anamorphic factor

\[
\begin{align*}
m_t &= \frac{n_1 \cdot u_{t,1}}{n_k \cdot u_{t,k}} \\
m_s &= \frac{n_1 \cdot u_{s,1}}{n_k \cdot u_{s,k}} \\
F_{anamoph} &= \frac{m_s}{m_t}
\end{align*}
\]
Realization of an anamorphic imaging with cylindrical lenses
Anamorphic Systems

- Transforming a circular into a rectangular image format
- Astigmatism over the field of view
Curved Object Surface

- Object surface is spherical bended with radius R:
  - image is bended by R'
- Paraxial approximation:
  - depth transfer magnification gives
    \[ z = \frac{y^2}{2R} \]
    \[ \alpha = \frac{z'}{z} = -m^2 \]
    \[ R' = -\frac{R}{m} \]
- Notice: R and R' are bended with the same orientation,
  This behavior is opposite to the curved image in the Petzval picture
Comparison Imaging vs Illumination

- Imaging optics
  - point to point transfer
  - transfer of information

- Illumination
  - mapping extended source on extended target
  - imaging to be avoided
  - transfer of flux

- Comparison
  - different tasks
  - different tools
  - different methods

Ref.: J. Muschaweck
Illumination systems:

- Different requirements: energy transfer efficiency, uniformity
- Performance requirements usually relaxed
- Very often complicated structures components
- Problem with raytracing: a ray corresponds to a plane wave with infinity extend
- Usual method: Monte-Carlo raytrace
  Problems: statistics and noise
- Illumination systems and strange components needs often a strong link to CAD data
- There are several special software tools, which are optimized for (incoherent) illumination:
  - LightTools
  - ASAP
  - FRED
Photometric Properties

- Relations of the 4 main definitions
- Cassarly's diamond

Ref.: J. Muschaweck
### Radiometric vs Photometric Units

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<th>Formula</th>
<th>Radiometric</th>
<th>Photometric</th>
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<tr>
<td><strong>Term</strong></td>
<td><strong>Unit</strong></td>
<td><strong>Term</strong></td>
<td><strong>Unit</strong></td>
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<tr>
<td>Energy</td>
<td>Energy</td>
<td>Ws</td>
<td>Luminous Energy</td>
</tr>
<tr>
<td>Power</td>
<td></td>
<td>W</td>
<td>Luminous Flux</td>
</tr>
<tr>
<td>Radiation flux</td>
<td>$\Phi$</td>
<td>Radiance</td>
<td><strong>W / sr</strong></td>
</tr>
<tr>
<td>Power per area and solid angle</td>
<td>$L = \frac{d^2 \Phi}{\cos \theta d\Omega dA}$</td>
<td>Radiant Intensity</td>
<td><strong>W / m$^2$/sr</strong></td>
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<tr>
<td>Emitted power per area</td>
<td>$E = \frac{d\Phi}{dA} = \int L \cos \theta d\Omega$</td>
<td>Radiant Excitance</td>
<td><strong>W / m$^2$</strong></td>
</tr>
<tr>
<td>Incident power per area</td>
<td>$E = \frac{d\Phi}{dA} = \int L \cos \theta d\Omega$</td>
<td>Irradiance</td>
<td><strong>W / m$^2$</strong></td>
</tr>
<tr>
<td>Time integral of the power per area</td>
<td>$H = \int E , dt$</td>
<td>Radiant Exposure</td>
<td><strong>Ws / m$^2$</strong></td>
</tr>
</tbody>
</table>
Differential Flux

- Differential flux of power from a small area element \(dA_s\) with normal direction \(n\) in a small solid angle \(d\Omega\) along the direction \(s\) of detection

\[
d^2\Phi = L \cdot d\Omega \cdot dA_{s\perp} = L \cos \theta_s \cdot d\Omega \cdot dA_s = L \cdot d\Omega \left( \vec{s} \cdot d\vec{A}_s \right)
\]

\(L\) radiance of the source

- Integration of the radiance over the area and the solid angle gives a power
Radiance independent of space coordinate and angle

The irradiance varies with the cosine of the incidence angle

Integration over half space

Integration of cone

Real sources with Lambertian behavior:
black body, sun, LED

\[ L(\vec{r}, \vec{s}) = L = \text{const} \]

\[ E(\theta) = L \cdot A \cdot \cos \theta = E_o \cdot \cos \theta \]

\[ \Phi_{Lam}^{HR} = \int E(\theta) \cdot d\Omega = \pi \cdot A \cdot L \]

\[ \Phi_{Lam}(\varphi) = \pi AL \cdot \sin^2 \varphi \]
Fundamental Law of Radiometry

- Differential flux of power from a small area element $dA_S$ on a small receiver area $dA_R$ in the distance $r$, the inclination angles of the two area elements are $\theta_S$ and $\theta_R$ respectively.

Fundamental law of radiometric energy transfer

$$d^2\Phi = \frac{L}{r^2} \cdot dA_{S\perp} dA_{E\perp}$$

$$= \frac{L}{r^2} \cdot \cos \theta_S \cos \theta_E dA_S dA_E$$

- The integration over the geometry gives the total flux
Basic task of radiation transfer problems: integration of the differential flux transfer law

\[ d^2 \Phi = \frac{L}{r^2} \cdot dA_s \cdot dA_e \]

Two classes of problems:
1. Constant radiance, the integration is a purely geometrical task
2. Arbitrary radiance, a density function has to be integrated over the geometrical light tube

Special cases:
Simple geometries, mostly high symmetric, analytical formulas

General cases: numerical solutions
- Integration of the geometry by raytracing
- Considering physical-optical effects in the raytracing:
  1. absorption
  2. reflection
  3. scattering
### General setup

**Optical System**

- **Radiation Source**
  - $dA_S$
  - $P_S$
- **Optical System**
  - $\Phi_{in}$
  - $\Phi_{out}$
- **Detector**
  - $dA_D$
  - $P_D$

**Ref:** B. Dörband
Transfer of Energy in Optical Systems

- Conservation of energy
- Differential flux
- No absorption
- Sine condition fulfilled

\[ d^2 \Phi = d^2 \Phi' \]
\[ d^2 \Phi = L \cdot \sin u \cdot \cos u \cdot dA \cdot du \cdot d\varphi \]
\[ T = 1 \]
\[ n \cdot y \cdot \sin u = n' \cdot y' \cdot \sin u' \]
Natural Vignetting: Setup with Rear Stop

- Stop behind system: exact integration possible

\[ E(w') = \frac{\pi \cdot L}{2} \cdot \left( \frac{n'}{n} \right)^2 \cdot \left[ 1 - \left( 1 + \frac{4 \cos^2 w' \cdot \tan^2 u'}{(1 - \cos^2 w' \cdot \tan^2 u')^2} \right)^{-1/2} \right] \]

- Special case on axis

\[ E'(0) = \pi L' \sin^2 u' = \left( \frac{n'}{n} \right)^2 \cdot \pi \cdot L \sin^2 u' \]

- Approximation small aperture: Classical cos-to-the-fourth-law

\[ E(w') = E(0) \cdot \cos^4 w' \]
Real Systems: Vignetting

- Artificial vignetting by truncation of rays
- Change of usable pupil area due to lens diameters, stops,...
- Approximation for uniform illuminated pupils: irradiance decreases proportional to effective pupil area